The 99 KPPY Combinatorics Seminar

Organized by S. Bang, J. Park, and M. Siggers

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Yeungnam University

Science Building 1 (F21), Room 205

Program

11:30 - 12:10 **Richard Stanley** University of Miami
Some combinatorial aspects of cyclotomic polynomials

12:10 **Lunch**

1:30 - 2:10 **Jang Soo Kim** SKKU
Lecture hall graphs and the Askey scheme

2:20 - 3:00 **Hongjun Ge** University of Science and Technology of China
A Bose-Laskar-Hoffman theory for μ-bounded graphs with fixed smallest eigenvalue

3:20 - 4:00 **Hyungtae Baek** Kyungpook National University
The Anderson rings over a von Neumann regular ring

4:10 - 4:50 **Mathias Schacht** University of Hamburg
Extremal problems for uniformly dense hypergraphs

5:00 **Banquet**

Abstracts

Richard Stanley

Some combinatorial aspects of cyclotomic polynomials

Euler showed that the number of partitions of n into distinct parts is equal to the number of partitions of n into odd parts. MacMahon showed that the number of partitions of n for which no part occurs exactly once is equal to the number of partitions of n into parts divisible by 2 or 3. Both these results are instances of a general phenomenon based on the fact that certain polynomials are the product of cyclotomic polynomials. After discussing this assertion, we explain how it can be extended to such topics as counting certain polynomials over finite fields and obtaining Dirichlet series generating functions for certain classes of integers.

Jang Soo Kim

Lecture hall graphs and the Askey scheme

We establish, for every family of orthogonal polynomials in the Askey scheme and the q-Askey scheme, a combinatorial model for mixed moments and coefficients in terms of paths on the lecture hall lattice. This generalizes to all families of orthogonal polynomials in the Askey scheme previous results of Corteel and Kim for the little q-Jacobi polynomials. This is joint work with Sylvie Corteel, Bhargavi Jonnadula, and Jon Keating.

Hongjun Ge

A Bose-Laskar-Hoffman theory for μ-bounded graphs with fixed smallest eigenvalue

In 2018, by using Ramsey and Hoffman theory, Koolen, Yang, and Yang gave a structural result on graphs with smallest eigenvalue at least -3 and large minimum degree. Without using Ramsey theory, we combine Bose-Laskar type argument and Hoffman theory to show some structural results about

μ-bounded graphs with fixed smallest eigenvalue. A consequence is that we have a reasonable bound for the minimum degree. Note that local graphs of distance-regular graphs is μ-bounded. We apply these results to describe the structure for any local graph of a distance-regular graph with classical parameters (D, b, α, β). As a consequence, we give a bound on α in terms of b. In particular, we show that $\alpha \leq 2$ if $b = 2$ and $D \geq 12$. This is joint work with J. Koolen, C. Lv, and Q. Yang.

Hyungtae Baek

The Anderson rings over a von Neumann regular ring

Many ring theorists researched various properties of Nagata rings and Serre's conjecture rings. In this talk, we introduce a subring (refer to the Anderson ring) of both the Nagata ring and the Serre's conjecture ring (up to isomorphism), and investigate properties of the Anderson rings. Additionally, we compare the properties of the Anderson rings with those of Nagata rings and Serre's conjecture rings. More precisely, we examine the following questions:

- When is the Anderson ring a principal ideal ring?
- When is the Anderson ring a Prüfer-like ring?

Mathias Schacht

Extremal problems for uniformly dense hypergraphs

Extremal combinatorics is a central research area in discrete mathematics. The field can be traced back to the work of Turán and it was established by Erdős through his fundamental contributions and his uncounted guiding questions. Since then it has grown into an important discipline with strong ties to other mathematical areas such as theoretical computer science, number theory, and ergodic theory.

We focus on extremal problems for hypergraphs, which were introduced by Turán. After solving the analogous question for graphs, Turán asked to determine the maximum cardinality of a set E of 3-element subsets of a given n-element set V such that for any 4 elements of V at least one triple is missing in E. This innocent looking problem is still open and despite a great deal of effort over the last 80 years and our knowledge is still somewhat limited. We consider a variant of the problem by imposing additional restrictions on the distribution of the 3-element subsets in E. These additional assumptions yield a finer control over the corresponding extremal problem. In fact, this leads to many interesting and more manageable subproblems, some of which were already considered by Erdős and Sós in the 1980ies. The additional assumptions on the distribution of the 3-element subsets are closely related to the theory of quasirandom discrete structures, which was pioneered by Szemerédi and became a central theme in the field. In fact, the hypergraph extensions by Gowers and by Rödl et al. of the regularity lemma provide essential tools for this line of research.