# The 33rd PNU-PMI Algebraic Combinatorics Seminar

Organized by M.Hirasaka and J.Koolen

September 19, 2009

#### Date

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## Place

Mathematics Science Building Room 404, POSTECH

## Program

11:00–11:50, Mark Siggers (Kyungpuk National University) Universal Algebra and Constraint Satisfaction

13:50–14:40, Yin-huan Han (Pusan National University) Convergence of Newton's method for solving a matrix polynomial

14:50–15:40, Meang Sang Park (Pusan National University) Braid presentation of a knotted arc with singular points

16:00–16:50, Shao Fei Du (Capital normal University and POSTECH) Graph Coverings and Lifting Techniques

17:00–17:30, Mitsugu Hirasaka (PNU) and Jack Koolen (POSTECH) Open Problem Session

## Available Devices for Presentation

We strongly encourage speakers to give a classical styled talk with chalk and blackboard. However, one beam projector is equipped at the room.

Speaker: Mark Siggers (Kyunguk National University) Title: Universal Algebra and Constraint Satisfaction Abstract: In about 2000, Jeavons popularised the use of universal algebra in the study of graph homomorphism complexity problems. We introduce the basics of universal algebra, descibe the interplay between it and graph homomorphisms, and then survey advances towards the Constraints Satisfaction Dichotomy conjecture which take advantage of this this interplay.

Speaker: Yin-huan Han (Pusan National University) Title: Convergence of Newton's method for solving a matrix polynomial. Abstract: We consider matrix polynomial which has the form

$$P(X) = A_0 X^m + A_1 X^{m-1} + \dots + A_m = 0$$

where X and  $A_1, A_2, \cdots, A_m$  are  $n \times n$  matrices with complex elements. For solving this matrix equation Newton's method was considered by Kratz and Stickel, Berlloulli's iteration was suggested by Dennis, Jr., Traub and Weber. See and Kim incorporated the exact line searches into Newton's method to reduced the number of iterations. They also improved the approach to solve each Newton step by Schur algorithm. In this work, we also drive Newton's method to solve matrix polynomial. We know each step of Newton's method for solving matrix equation involves finding the solution  $H \in \mathbb{C}^{n \times n}$  of the linear equation

(1) 
$$\sum_{i=1}^{m} \left\{ \left( \sum_{j=0}^{m-i} A_j X^{m-(j+i)} \right) H X^{i-1} \right\} = -P(X)$$

which can be rewrite as

(2) 
$$\sum_{i=1}^{m} B_i H C_i = D,$$

where  $B_i = \sum_{j=0}^{m-i} A_j X^{m-(j+i)}$ ,  $C_i = X^{i-1}$  and D = -P(X). Thus, we solve the linear equation (1), the matrix polynomial is automatically solved. Such an equation which has a similar form with the equation above has been studied. Mao et al presented an efficient iterative method for finding the generalized centro-symmetric solution of matrix equation AXB = C, where  $A, B, C \in \mathbb{R}^{n \times n}$ . Peng presented an iterative method for solving equation  $\sum_{i=1}^{l} A_i X_i B_i = C$  over bisymmetric matrix group  $[X_1, X_2, \cdots, X_l]$ . They have been proved that the iteration method can be terminated within finite iteration steps for any initial matrix. We suggest an iterative algorithm to solve Newton steps with symmetric and generalized centro-symmetric(res. bisymmetric) solution. In addition, the convergence of Newton's method with iterative algorithm for solving matrix polynomial is discussed. Finally, we give some numerical experiments. This is joint work with Hyun-Min Kim

Speaker: Meang Sang Park (Pusan National University) Title: Braid presentation of a knotted arc with singular points. Abstract: A knotted arc with singular points is an immersed image of an unit interval into  $S^3$  with finite transversal double singular points such that the images of two endpoints of the unit interval are different and not the singular points. In this talk we consider braid presentation of a knotted arc with singular points.

Speaker: Shao Fei Du (Capital Normal University and POSTECH) Title: Graph coverings and lifting techniques Abstract: In this talk, I shall present some methods for constructing voltage graphs by combinatorial tools and by group theoretical tools, and show a linear criteria for liftings of automorphisms when the covering transformation group is elementary abelian. As an application, I shall show a classication of 2-arc-transitive regular covers of complete graphs with the covering transformation group is  $\mathbf{Z}_p^3$  and a classication of 2-arc-transitive Cayley graphs on dihedral groups.