

The 31st PNU–PMI Algebraic Combinatorics Workshop

Organized by M.Hirasaka and J.Koolen

June 10, 2009

Date

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Place

Mathematics Science Building Room 404, POSTECH

Program

11:00–11:50, Ebrahim Ghorbani (POSTECH and Sharif University of
Technology, Iran)

Energy, chromatic number, and choice number of graphs

13:50–14:40, Hiroshi Suzuki (International Christian University)

The Terwilliger Algebra of a Polynomial Space

14:50–15:40, Shintar Kuroki (KAIST)

On extended G -actions of torus manifolds

16:00–16:50, Joonkyung Lee (KAIST)

The Rank of Skew–Symmetric Random Matrices Over Finite Fields

17:00–17:50, Dong Yeol Oh (POSTECH)

Formal power series over \mathbb{F}_q -domain

Available Devices for Presentation

We strongly encourage speakers to give a classical styled talk with chalk and blackboard. However, one beam projector is equipped at the room.

Speaker: Ebrahim Ghorbani (POSTECH and Sharif University of Technology, Iran)

Title: Energy, chromatic number, and choice number of graphs

Abstract: The energy of a graph G , denoted by $E(G)$, is defined as the sum of the absolute values of all adjacency eigenvalues of G . The talk is mainly about some bounds for chromatic number and choice number of graphs in terms of graph energy. More precisely, we show that apart from a few families of graphs,

$$E(G) \leq 2 \max(\text{ch}(G), n - \overline{\chi}(G)),$$

where $\overline{\chi}(G)$, $\chi(G)$, and $\text{ch}(G)$ are the complement, the chromatic number, and the choice number of G , respectively.

Speaker: Hiroshi Suzuki (International Christian University)

Title: The Terwilliger Algebra of a Polynomial Space

Abstract: In 70s Delsarte developed an algebraic theory of a subset Y of the base set X of a symmetric association scheme $X = (X, \{R_i\}_{0 \leq i \leq d})$. His theory was successfully applied to design theory and coding theory. In 90s Terwilliger defined the subconstituent algebra of an association scheme, which is also called the Terwilliger algebra. The Terwilliger algebra was successfully applied mainly to P - and Q -polynomial association schemes. Recently in a paper titled "Width and dual width of subsets in polynomial association schemes", Brouwer, Godsil, Koolen and Martin studied the theory of a subset of the base set of a P - and/or Q -polynomial association scheme from a view point different from Delsarte.

In this talk, we define the Terwilliger algebra of a polynomial space consisting of a Hermitian matrix $A \in \text{Mat}(C)$ and an orthogonal direct sum decomposition of the Hermitian space $V = C^n$ into subspaces V_0, V_1, \dots, V_t satisfying

$$AV_i \subseteq V_{i-1} + V_i + V_{i+1}, \text{ for all } i \in \{0, 1, \dots, t\}$$

with $V_{-1} = V_{t+1} = 0$. We develop basic theory and introduce results on thin

irreducible modules and the shortest modules. These results give a partial generalization of the theory developed by Brouwer, Godsil, Koolen and Martin.

Speaker: ShintarKuroki (KAIST)

Title: On extended G -actions of torus manifolds

Abstract: A torus manifold is a $2n$ -dimensional manifold with half dimensional torus actions with non-empty fixed points. This notion was introduced by Hattori and Masuda in 2003 as an ultimate generalization of toric manifolds. In this talk, we consider extended actions of these torus actions. In particular, we talk about a classification of extended actions with codimension zero and one principal orbits.

Speaker: Joonkyung Lee (KAIST)

Title: The Rank of Skew-Symmetric Random Matrices Over Finite Fields

Abstract: Let a_n be the probability an $2n$ by $2n$ random skew-symmetric matrix over the finite field $\text{GF}(q)$ is nonsingular, in which each entry is chosen uniformly at random from $\text{GF}(q)$. Carlitz (1954) proved that a_n converges to $(1 - q^{-1})(1 - q^{-3})(1 - q^{-5})\dots$ as n goes to infinity. This theorem has several consequences; for instance, a random graph with an even number of vertices would have an odd number of perfect matchings with the probability converging to about 42 %. We present two additional proofs for the above theorem. One proof is based on combinatorial arguments, and the other proof is based on Markov chains and their stationary distributions. Our new method provides further nontrivial generalizations. This is a joint work with Sang-il Oum.

Speaker: Dong Yeol Oh

Title: Formal power series over π -domain

Let R be an integral domain, X be a set of indeterminates over R , and $R[[X]]_3$ be the full ring of formal power series ring in X over R . An integral domain is called a π -domain if every principal ideal is a product of prime ideals. An integral domain R is called a formally stable π -domain if $R[[X_1, \dots, X_n]]$ is a π -domain for each finite set $\{X_1, \dots, X_n\}$ of indeterminates over R . We show that R is a formally stable π -domain if and only if $R[[X]]_3$ is a π -domain. We extend the result to rings with zero-divisors. A commutative ring R with identity is called a π -ring if every principal ideal is a product of prime ideals. We show that R is a formally stable π -ring if and only if $R[[X]]_3$ is a π -ring.