The 28th PNU−POSTECH Algebraic Combinatorics Workshop

Organized by M.Hirasaka and J.Koolen

November 15, 2008

Date November 15, 2008
Place C32−211, Department of Mathematics in Pusan National University
Program

10:30−11:20, Suyoung Choi (KAIST)
Combinatorics on bigraded Betti numbers of simple
polytopes
11:30−12:20, Hidefumi Kawasaki (Kyushu University)
Discrete xed point theorems and their applications to the game theory

14:00−14:50, Kyoung Ho Park (Kyungpook National University),
On the distribution of Genocchi polynomials

15:00−15:50, Jungwook Lim (POSTECH),
Zero−divisor graphs of polynomials and power series over commutative rings

16:10−17:00, Soohak Choi (POSTECH),
The new lower bounds on covering arrays

17:10−18:00, Jong Yoon Hyun (POSTECH),
The isometry group of an arbitrary poset−metric space

19:00−21:00, Banquet (free of charge)

Available Devices for Presentation
We strongly encourage speakers to give a classical styled talk with chalk and blackboard. However, one beam projector is equipped at C32−208.

Important Notice In principle, each participant has to give comments or questions at least twice during the seminar.

Speaker: Suyoung Choi

Title: Combinatorics on bigraded Betti numbers of simple polytopes

Abstract: A polytope P is called simple if there are exactly n facets meeting at each vertex of P. Assume that P has m facets. The main object of this talk is the bigraded Betti numbers which appear in a minimal free resolution of face ring of a simple polytope P over a polynomial ring with m variables. These numbers are quite related to the combinatorics of simple polytopes including the sum of connected components for all choices of k facets in P , denote $c_k(P)$. One of the most important remarks is the following; the $(k-1; k)$−th Betti number $b_k(P)$ of P is

$$b_k(P) = c_k(P) + \binom{m}{k} \quad : \quad m3$$

In this talk, we established the formula of b_k of a connected sum of simple polytopes P and Q. The formula shows that $b_k(P\#Q)$ is dependent only on P and Q but the base eld k and how they connected. Using this, we can compute purely combinatorially the b_k of stacked polytopes of dimension n and prisms of dimension 3.

On the other hand, we investigate the maximality of bigraded Betti numbers of 3−dimensional stacked polytopes and prisms. A polytope is called irreducible if it can not be represented by a connected sum of several polytopes.

Problem Let P be a polytope of dimension 3 with m facets and let $_{m-3}$ and P(m−2) be a stacked 3−polytope and a 3−prism which have m facets, respectively. Then, for any k, we have the inequality

$$b_k(P) < \begin{cases} b_k(P(m-2); & \text{if } P \text{ is irreducible;} \\ b_k(_{m-3}); & \text{otherwise.} \end{cases}$$

We give a non−trivial armative solution of Problem ?? for k = m−4. Also we prove that the equality holds if and only if P is a such polytope. As a corollary, we prove that every prism is cohomologically rigid. This work is jointly with Jang Soo Kim.

Speaker: Hidefumi Kawasaki (Kyushu University)
Title: Discrete xed point theorems and their applications to the game theory

Abstract: In bimatrix game, there are two players P_1 and P_2. Player P_1 has m choices (pure strategies), and player P_2 has n choices. When P_1 and P_2 respectively choose i−th pure strategy and j−th pure strategy, they respectively gain a_{ij} 2 R and b_{ij} 2 R, and each player maximize his/her gain. Here, matrices A := (a_{ij}) and B := (b_{ij}) are called payoff matrices.

In order to find a solution (equilibrium, strategies) that both players satisfy, we need mixed strategies, that is, both players throw dice to decide their strategies. Then the bimatrix game is formulated as follows:

$$(P_1) \quad \max_{x 2 S_m} x^T A y; \qquad (P_2) \quad \max_{y 2 S_n} x^T B y;$$

where S_m := {x = (x_1; : : : ; x_m) 2 R^m ; x 0 8i; x_1 = 1}, and there exist x 2 S_m and y 2 S such that

$$x^T A y < x^T A y \ 8x \ 2 \ S_m \qquad x^T B y \quad x^T B y \ 8y \ 2 \ S_n$$

Such a pair (x; y) is called a Nash equilibrium. Nash proved his claim by Brouwer's fixed point theorem: Any continuous mapping from a compact convex set C R^n into itself has a fixed point.
The aim of this talk is to introduce the recent developement of discrete xed point theorems and their applications to the game theory.

Speaker: Kyoung Ho Park (Kyungpook National University)
Title: On the distribution of Genocchi polynomials

Abstract: In this talk, we introduce Genocchi numbers and polynomials. Firstly, we study the distribution of Genocchi polynomials. Secondly, we investigate the symmetry for the distribution of twisted q−Genocchi numbers and polynomials associated with the fermionic p−adic invariant integral on Z

Speaker: Jungwook Lim
Title: Zero−divisor graphs of polynomials and power series over commutative rings

Abstract: Let R be a commutative ring with identity and Z(R) (resp. Z(R) the set of zero− divisors(resp. nonzero zero−divisors) of R. By the zero−divisor graph of R, denoted by (R), we mean the graph whose vertices are the nonzero zero−divisors of R, and for distinct r; s 2 Z(R), there is an edge connecting r and s if and only if rs = 0. In this talk, I will talk about diameter and girth of (R); (R[X]) and (R[[X]]).

Speaker: Soohak Choi
Title: The new lower bounds on covering arrays
Abstract: Let $B_q = \{0; 1; : : : ; q-1\}$ be a set with q elements. An $m * n$ matrix C over B_q is called a t−covering array (or, a covering array of size m, strength t, degree n, and order q) if, in any t columns of C, all q_t possible q−ary t−vectors occur at least once. It is one of the main problem in the theory of covering arrays to find the minimum size $g_t(n)$ of a t−covering array of given degree n. The main problem was completely solved only for the case t = 2 and q = 2. Roux gives two useful bounds.

$g_{t+1}(n + 1)$ $2g_t(n)$; $g_3(2n)$ $g_3(n) + g_2(n)$: We will give the better bound.

Speaker: Jong Yoon Hyun (POSTECH),
Title: The isometry group of an arbitrary poset−metric space
Abstract: In this talk, I give a complete description of isometries of an arbitrary poset−metric space and present the structure of the isometry group as well as its size. The computation of its size for a special type of poset−metric spaces which were well−studied in the literature is also given.