The 47th Combinatorics Seminar

Organized by S.Bang, M.Hirasaka, T.Jensen, J.Koolen and M.Siggers

September 03, 2011

Place

Department of Mathematics in Yeungnam University
Science Building 1, Room 205

Program (September 03, 2011)

11:00–11:50, Sandi Klavžar (Univ. of Ljubljana, Univ. of Maribor and IMFM, Slovenia)
Domination Game Played on Graphs

12:00– , Lunch

13:30–14:20, Lee Jung Yun (KIAS)
Todd operator, special values of zeta function and generalized Dedekind sum

14:30–15:20, Kim, Ji Young (Seoul National University)
The Kloosterman problem for Hermitian lattices

15:40–16:30, Hyonju Yu (POSTECH)
Some construction of regular graphs

16:40–17:30, Kim Kyoung-tark (Pusan National University)
On decompositions of $\mathfrak{sl}(n, F)$ over a field of positive characteristic.

18:00–, Dinner

Speaker: Sandi Klavžar (University of Ljubljana, University of Maribor and IMFM, Slovenia)
Title: Domination Game Played on Graphs
Abstract: In this talk a domination game (introduced in B. Brešar, S. Klavžar, D.F. Rall, Domination game and an imagination strategy, SIAM J. Discrete Math. 24 (2010) 979–991) will be presented. Just like in the classical coloring game variant, we have a player, called Dominator, who wishes to dominate a graph in as few steps as possible and another player, called Staller, who wishes to delay the process as much as possible. The players alternate their moves and vertices must be chosen in such a way that whenever a vertex is chosen by either player, at least one additional vertex of the graph G is dominated that was not dominated by the vertices previously chosen. There are two different games, in Game 1 Dominator starts the game while in Game 2 Staller has the first move. The *game domination number* $\gamma_g(G)$ of a graph G is the total number of vertices chosen when Game 1 is played on G using optimal strategies by both player. Similarly, the *Staller-start game domination number* $\gamma'_g(G)$ of G, is the cardinality of the set of vertices chosen when Game 2 is played on G.
An overview of what is know about this domination game will be given. In particular, $\gamma_g(G)$ and $\gamma'_g(G)$ can differ by at most one (proved in part in a manuscript by Bill Kinnersley and his co-workers), and most of possible pairs $(\gamma_g(G), \gamma'_g(G))$ can be realized on trees. Bounds on $\gamma_g(G)$ will be given for general graphs and for particular classes of graphs. The domination game will also be connected with Vizing's conjecture. Numerous open problems and conjectures will also be stated.

Speaker: Lee Jung Yun (KIAS)
Title: Todd operator, special values of zeta function and generalized Dedekind sum
Abstract: Using Todd operator one can express Euler-McLaurin formula in simple form. For recent decades, along with development of toric geometry and theory of polytopes, Euler-McLaurin formula has been generalized to a category of lattice cones and polytopes by M. Brion, M. Vergne, S. Garoufalidis, J. Pommersheim and several others. In particular, Garoufalidis and J. Pommersheim expressed special values of zeta function associated to Todd operator associated to a certain cone decomposition. This is later applied to find certain type of reciprocity law of generalized Dedekind sums. We extend the category of cones by Grothendieck group construction of ordinary cones. This new category contains 'virtually decomposed cones' considering cones with negative weight and the appropriate form of Todd operator construction

generalizes the Euler-Mclaurin formula on it. We apply this generalization to obtain an alternating sum expression of special values of (partial) zeta functions at nonpositive integers associated (virtual) cone decomposition. This is much simpler than that obtained by Garoufalidis and Pommersheim. It is joint work with Byungheup Jun.

Speaker: Kim Ji Young (Seoul National University)
Title: The Kloosterman problem for Hermitian lattices
Abstract: A positive definite Hermitian lattice over an imaginary quadratic field $\mathbb{Q}(\sqrt{-m})$ is called almost universal if it represents all sufficiently large positive integers. We investigate almost universal binary Hermitian lattices and find a well-organized simple characterization of all almost universal binary Hermitian lattices over all imaginary quadratic fields. Especially we prove if p is odd or $p = 2$ and $m \equiv 3 \pmod 4$, then all almost universal p-anisotropic binary Hermitian lattices over $\mathbb{Q}(\sqrt{-m})$ are universal, and give the complete list of all such Hermitian lattices.

Speaker: Hyonju Yu (POSTECH)
Title: Some construction of regular graphs
Abstract: For a given graph G and a positive integer r, we construct $(d_{max}(G) + r - 1)$-regular graph $G(r)$ such that $\lambda_{min}(G(r)) \in [\lambda_{min}(G) - 1, \lambda_{min(G)}]$ and $\lim_{r\to\infty} \lambda_{min}(G(r)) = \lambda_{min}(G) - 1$. As applications, we obtains following theorems.

Theorem 1 *Let γ_k^{reg} be the supremum of the smallest eigenvalues of k-regular graphs with smallest eigenvalue < -2. Then $\lim_{k\to\infty} \gamma_k^{reg} = -1 - \sqrt{2}$.*

Theorem 2 *Let δ_k^{reg} be the supremum of the smallest eigenvalues of k-regular graphs with smallest eigenvalue $< -1 - \sqrt{2}$. Then $\lim_{k\to\infty} \gamma_k^{reg} = \delta$ where δ is the smallest root(≈ -2.4812) of the polynomial $x^3 + 2x^2 - 2x - 2$.*

Theorem 3 *Every number in the interval $(-\infty, -\sqrt{2 + \sqrt{5}} - 1]$ is a limit point of the smallest eigenvalues of regular graphs.*

Speaker: Kim Kyoung-tark (Pusan National University)
Title: On decompositions of $\mathfrak{sl}(n, F)$ over a field of positive characteristic.
Abstract: It is well-known that for each prime power n the special linear algebra $\mathfrak{sl}(n, \mathbb{C})$ has an OD(orthogonal decomposition) into CSAs(Cartan subalgebras) with respect to the Killing form. But, it is still open whether

$\mathfrak{sl}(n, \mathbb{C})$ has such an OD unless n is a prime power. So, it is natural to ask what happens if we consider $\mathfrak{sl}(n, F)$ with $\mathrm{char} F > 0$. In this modular case, $\mathfrak{sl}(n, F)$ is a restricted Lie algebra whose name and concept are first introduced by N. Jacobson. In a restricted Lie algebra L, a *torus* J is an abelian p-subalgebra of L such that J_K has no nonzero p-nilpotent elements where K is an algebraic closure of F and $J_K := K \otimes_F J$. There is a fact that a CSA is the centralizer of a maximal torus in a restricted Lie algebra, but we will show that a maximal torus is itself a CSA in $\mathfrak{sl}(n, F)$. We aim to know whether $\mathfrak{sl}(n, F)$ has a direct sum decomposition into maximal tori, and when components in the decomposition are pairwise orthogonal with respect to the trace form.