

# The 39th KNU-PNU-PMI Algebraic Combinatorics Seminar

Organized by M.Hirasaka, T. Jensen, J.Koolen and M. Siggers

May 1, 2010

## **Date**

May 1, 2010

## **Place**

Mathematics Science Building Room 404, POSTECH

## **Program**

11:00–11:50, Yoshio Sano (PMI)  
Matroids on convex geometries: Subclasses, operations, and optimization

12:00–13:45, Lunch break

13:50–14:40, Nayeong Kong (PNU)  
Introduction to Tropical Geometry and a Classification of the Tropical Conics

14:50–15:40, Woong Kook (University of Rhode Island, USA)  
Combinatorial Green's function of a graph and applications in networks

16:00–16:50, Matej Stehlik (National Sun Yat-sen University, Taiwan)  
Simultaneous colouring of plane graphs

17:00–17:50, Greg Markowsky (PMI)  
Random Walks and Electric Resistance on Distance-Regular Graphs

## **Available Devices for Presentation**

We strongly encourage speakers to give a classical styled talk with chalk and blackboard. However, one beam projector is equipped at the room.

Speaker: Yoshio Sano (PMI)

Title: Matroids on convex geometries: Subclasses, operations, and optimization

Abstract: A matroid-like structure defined on a convex geometry, called a cg-matroid, was introduced by S. Fujishige, G. A. Koshevoy, and Y. Sano [Matroids on convex geometries (cg-matroids), *Discrete Mathematics* **307** (2007) 1936–1950].

In this talk, we discuss the theory of cg-matroids. We first give the definition and examples of cg-matroids and then give some characterizations of cg-matroids by axioms. The axioms for independent sets (resp. spanning sets) of cg-matroids naturally lead a subclass of cg-matroids, called strict cg-matroids (resp. co-strict cg-matroids). These subclasses have nice properties.

After then, we consider operations on cg-matroids such as restriction and contraction. These operations are closely related to the subclasses of cg-matroids. We also consider an optimization problem on cg-matroids, which reveals the relation between the greedy algorithm and cg-matroids.

Speaker: Nayeong Kong (PNU)

Title: Introduction to Tropical Geometry and a Classification of the Tropical Conics

Abstract: I will introduce Tropical Geometry which is a relatively new area in algebraic geometry. It first appeared as a subject of its own in 2002. The name of tropical geometry has the following origin: The adjective "tropical" is given in honor of the Brazilian mathematician Imre Simon, who pioneered the field. The tropical semi-ring is the real numbers enhanced with infinity and equipped with two arithmetic operations called the tropical addition and the tropical multiplication. The tropical addition is the operation of taking minimum. The tropical multiplication is the conventional addition. Moreover, we outline a method for finding the combinatorial structures of hypersurfaces of tropical conics. Tropical conics are homogeneous polynomials of degree 2 in two variables. In tropical geometry, we can classify all tropical conics to finitely many kinds.

Speaker: Woong Kook (University of Rhode Island)

Title: Combinatorial Green's function of a graph and applications in networks

Abstract: Given a finite weighted graph  $G$  and its Laplacian matrix  $L$ , the *combinatorial Green's function*  $\mathcal{G}$  of  $G$  is defined to be the inverse of  $L + J$ ,

where  $J$  is the matrix each of whose entries is 1. We prove the following intriguing identities involving the entries in  $\mathcal{G} = (g_{ij})$  whose rows and columns are indexed by the vertices of  $G$ :

$g_{aa} + g_{bb} - g_{ab} - g_{ba} = \kappa(G_{a*b})/\kappa(G)$ , where  $\kappa(G)$  is the complexity or tree-number of  $G$ , and  $G_{a*b}$  is obtained from  $G$  by identifying two vertices  $a$  and  $b$ . As an application, we derive a simple combinatorial formula for the resistance between two *arbitrary* nodes in a finite resistor network.

Applications to information networks are also discussed.

Speaker: Matej Stehlik (National Sun Yat-sen University, Taiwan)

Title: Simultaneous colouring of plane graphs

Abstract: In a simultaneous colouring of a plane graph  $G$  we colour some or all of the elements (vertices, edges, and faces) of  $G$  so that adjacent or incident elements receive different colours. This concept was introduced by Ringel in 1965. The general problem is to determine the smallest  $k$  such that  $G$  can be simultaneously coloured with  $k$  colours. I will give an overview of the known results and open problems, and then present two results I recently obtained together with Ross Kang and Jean-Sebastien Sereni.

Speaker: Greg Markowsky (PMI)

Title: Random Walks and Electric Resistance on Distance-Regular Graphs

Abstract: There is a beautiful connection between random walks on graphs and electric resistance on circuits that I will discuss. I will discuss some recent work that Jack Koolen and I have been doing concerning these topics on distance-regular graphs.